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ESTIMATE OF THE DECREASE OF THE S FIELD WITH ALTITUDE

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Introduction

It can be interesting, with a view to satellite measurements, to attempt to estimate up to what altitude the field due to the diurnal variation S still has an important influence. /1*

The only rigorous procedure would, without doubt, be to start with data from a spherical harmonic analysis of the data from the earth.

A recent analysis by Price and Wilkins (ref. 1) shows, however, that the form of the current systems is in fact very complex and is rapidly deformed in the course of the day; the representation of altitude effects would therefore seem to be, a priori, extremely complex.

It can be useful to start with an extremely simplified scheme; its interest will be in giving orders of magnitude.

1. The Model

In order to estimate the altitude effects of the field S caused by the diurnal variation, we will start with a plane model consisting of two layers of circular currents (fig. 1), whose centers f are located 7000 km apart. We will study its effects in the perpendicular plane passing through the centers, a plane which is therefore representative of the effects of the field S in the vicinity of the noon meridian.

*Numbers given in the margin indicate the pagination in the original foreign test.

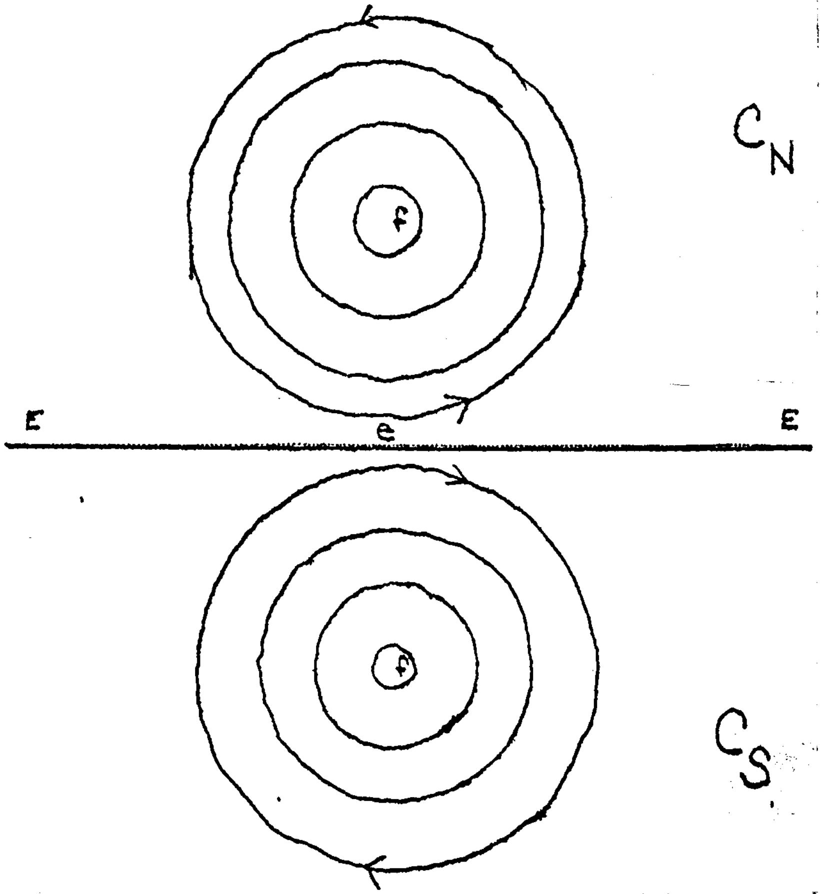

Figure 1. Model.

We will also vary the density of currents in each layer according to a parabolic law, from a value i = 1 to a value i = 0, between the line EE representing the equator to the center f; this law is similar to that proposed in reference 2, p. 232-3 for a simplified calculation of the current of the layers S obtained by means of a spherical harmonic analysis.

We will first try to estimate the approximations introduced by the choice of such a model by studying the relative variation of the field with respect to an altitude of 100 km above the plane as a function of different parameters.[1]

[1]In § 3 we will discuss the approximation made by the choice of a plane model.

From the point of view of the form and dimensions chosen to represent /2 the effect of current systems Sq around the meridian, it can be noted that such layers whose centers f should be located near ±35° latitude, or nearly at the mean latitude of the focus of the currents Sq, should have extended over only 70° of longitude rather than 180°. We will compare their effect to that of the layers formed by indefinite rectilinear currents, parallel to the line EE, and whose direction of the current should be reversed beyond ±3500 km from the line EE. The same law as for circular layers is used for the variation of current density.

Figure 2 represents in one part the decrease of the field with altitude (logarithmic abscissa scale) from an altitude of 100 km (where the field is supposed to 100 γ) along a vertical located at e (fig. 1), for these two models of circular or rectilinear currents.[1] This therefore involves a horizontal field, H.

The value of the field at each altitude has been obtained here, as in all that follows, by use of a planimeter on the curves representing the field

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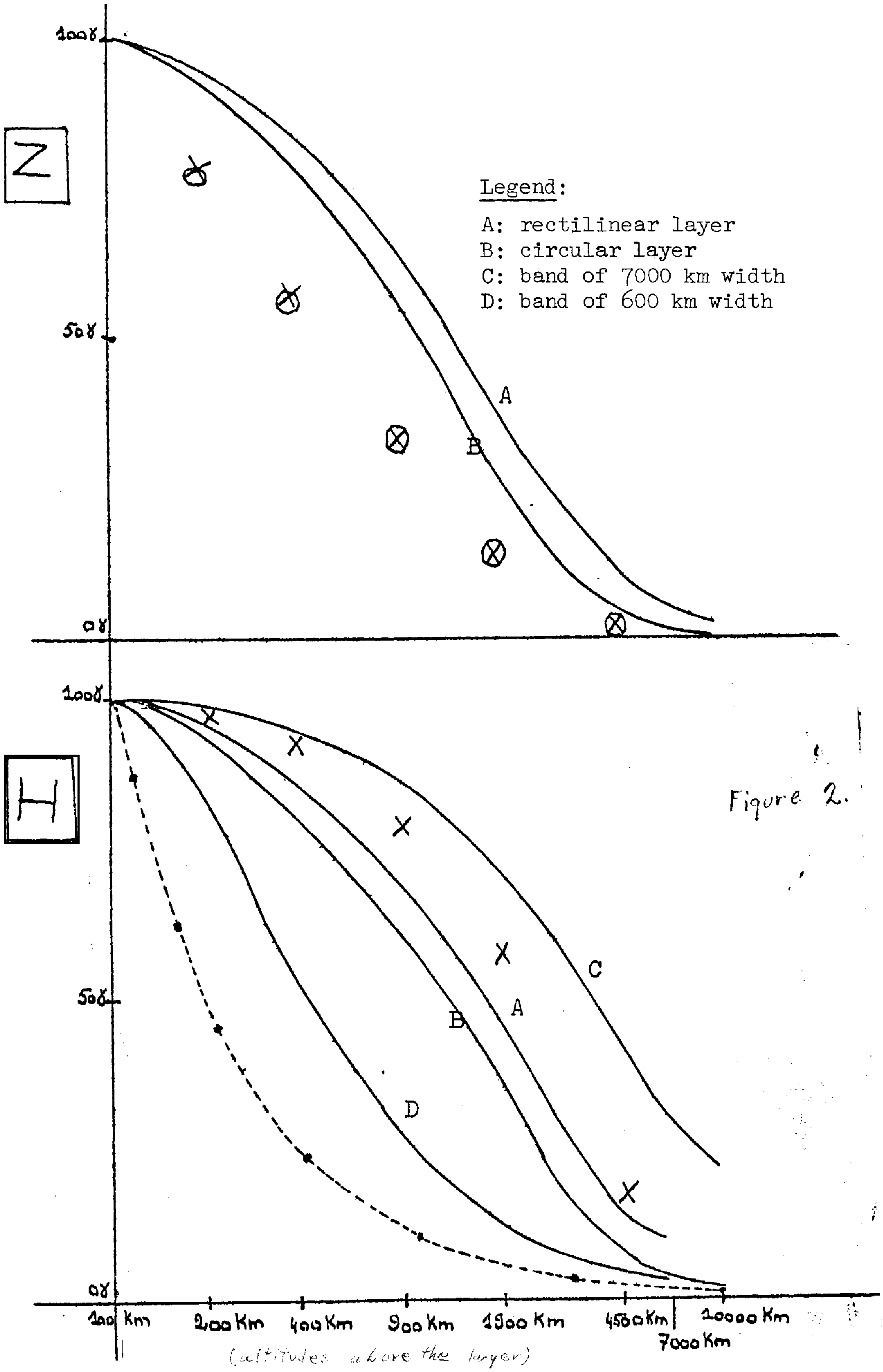
[1]The formulas giving the expression for the field for an element of indefinite rectilinear current are well known, as well as for a circular spiral at a point on its axis. For the calculation of the field away from the axis of a circular spiral, we have used expressions given in reference 3. They contain elliptical integrals, but their calculation is simple enough. In certain associated calculations, in particular for the case of layers of indefinite rectilinear currents of finite width and constant current density, we have used the mathematical expression of the integral, in the same way as for the field of circular layers of constant current density on the axis of symmetry.


Z
100γ
50γ
0γ
Legend:
A: rectilinear layer
B: circular layer
C: band of 7000 km width
D: band of 600 km width
A
B
H
100γ
50γ
0γ
C
A
B
D
100 Km
200 Km
400 Km
900 Km
1900 Km
4500 Km
7000 Km
10000 Km
(altitudes above the layer)
Figure 2.

intensity due to elements of current of the layer located at 500 km by steps of 500 km (in the case of very rapid variation, supplementary points were calculated).

In the other part of figure 2, the curves of the same quantity (Z) represent the decrease of the component perpendicular to the plane of the layers along the vertical located at f. In this preliminary case, the effect of the layer ϵ_S has been neglected.

Whether the layer is circular or linear, the law of decrease with /3
altitude has the same form entirely. This is why we shall take, in what follows, a model formed from two circular layers; in fact, a circular layer is undoubtedly closer to the real current system, which is rather elliptical, than a layer of rectilinear currents.

The approximation made by the choice of a parabolic law of current density variation can be estimated by a comparison with the effect of layers of the same form with constant current density. Thus, in figure 2, the symbols (x) represent, for H, the decrease of the field at e due to a rectilinear current with constant density (the direction of the current being reversed beyond 3500 km) and for Z, that at f of the field due to a single circular layer with constant current density. In the first case, the decrease is clearly slower than when the current density varies parabolically; it is clearly faster in the second case. In the case of a rectilinear layer of finite width, this difference is due to the fact that the effect of distant current elements, which is weak at low altitudes, is important at high altitudes. With a circular layer, on the other hand, the field is strongly increased at the center of the layer, at low altitudes by spirals of small radius, but the effect of these spirals decreases rapidly with altitude.

The choice of the parabolic law of current variation, which, according to reference 2, is valuable for estimating the intensity of real currents Sq, has therefore a real importance and we will keep it in our model.

We can still try to find out what approximation is introduced by taking a layer of zero thickness. To do this, starting with the integral for the value of the field on the axis of symmetry of a circular layer of radius 3500 km with constant current density, it should be possible to graphically integrate the values of the field of a layer of 50 km thickness by assum- /4 ing a constant current density throughout the thickness. In fact, when we study the curve of the variation of the integral of the field value, and the intervals in which the integration ought to be done, the curvature is weak enough for a very sufficient approximation to be given by considering three infinitely thin layers located at -25 km, 0 km and +25 km. The mean value of the fields produced by each of these three layers gives an approximation that is very valuable with respect to a layer of 50 km thickness. The result is shown by the symbols (o) on figure 2 (z); the difference with respect to a single layer with constant current is practically zero above 500 km; it scarcely reaches 1 or 2 percent at 400 and 200 km altitude. We can therefore consider the approximation of an infinitely thin layer to be quite valuable.

We know that the field created by a layer of rectilinear current of infinite width is a horizontal field of constant intensity as a function of altitude above the layer. Therefore, the law for the decrease of the field of circular (or rectilinear) layers whose dimensions are nearly the same as for a real system of currents Sq is profoundly different. Figure 3, which represents the value of the field between 10 and 1000 km altitude (logarithmic abscissa scale) at the center of a layer of rectilinear currents of 7000 km width with

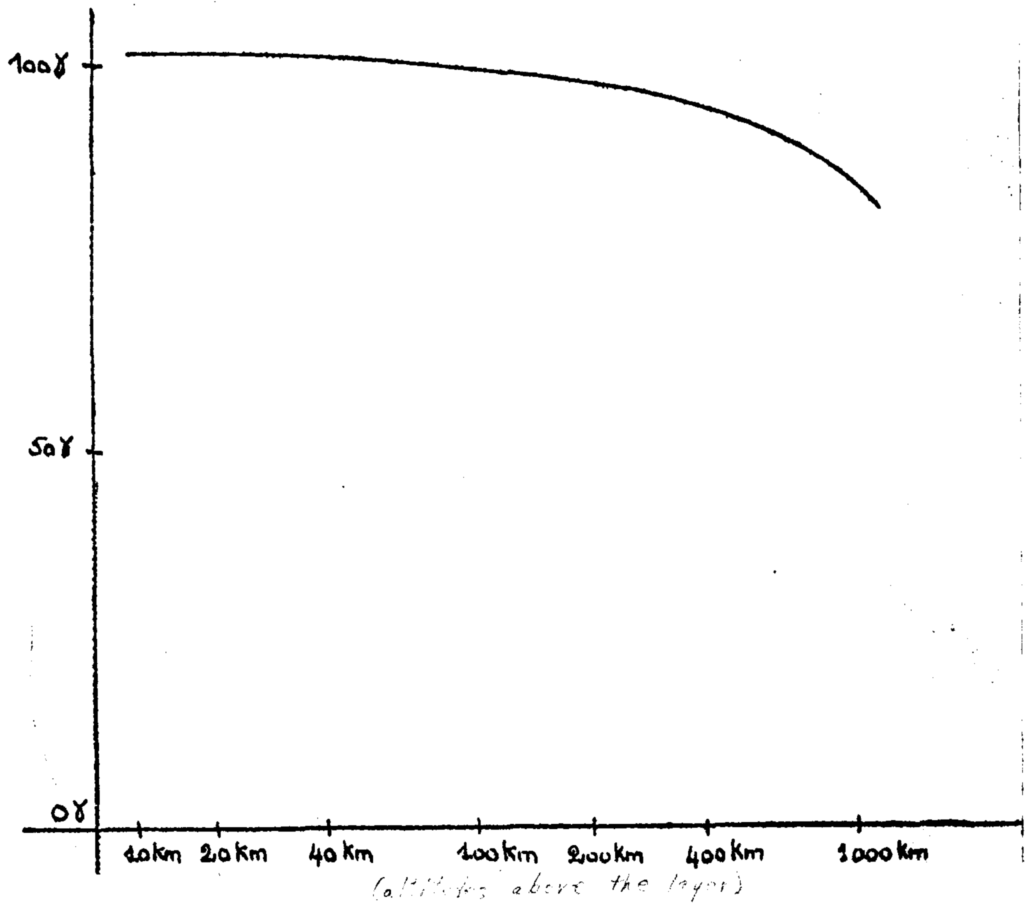


Figure 3

constant current density,[1] shows that the approximation of the layer of rectilinear currents of infinite width (constant field) should be valuable between the layer and an altitude of 100 km.

By way of comparison, there is also shown on figure 2 (H) the decrease with altitude of the field produced by an indefinite rectilinear current (dashed curve) and those produced at the center of bands of 600 km width (case

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[1] From there on, the difference above 100 km between the values of the field of this layer and those represented by the symbols (x) of figure 2 (H) for a rectilinear layer where the effect of a reversed current from 3500 to 7000 km was included.

of an electrojet) or 7000 km width. Thus, it seems clear enough how by starting with an indefinite rectilinear current and by increasing little by little the width of the band, the law of decrease inversely proportional to the distance from a rectilinear current is little by little deformed, tending, with a layer of infinite width, toward a constant field. However, this property of a field that is constant with altitude exists only at altitudes that are very low with respect to the width of the band. /5

Practically speaking, if we come back to our circular model with variable current density, we can roughly say that, from 100 to 7000 km altitude, the field decreases in a practically linear manner as a function of a logarithmic altitude scale, in the proportion 10 to 1. Beyond 10,000 km it becomes negligible.

These first remarks already give some idea of the probable limits beyond which the effect of the currents Sq become negligible. In fact, the decrease with altitude of the field Sq is profoundly different because it is necessary to consider the effect not of a single current layer, but the effects of an external layer and an internal layer.

2. Decrease with Altitude of the Field Produced by Two Parallel Systems of Circular Layers

We know that at the surface of the earth the effect of the system of induced currents is to increase the field S of the horizontal component, but to diminish that of the vertical component. This difference in behavior between the two components exists only in the space included between the two systems (fig. 4). The general effect of an induced current is, in fact, to oppose that of the inducing current. This is so, in all space for the component Z and, outside the space included between the two systems, for the component H.

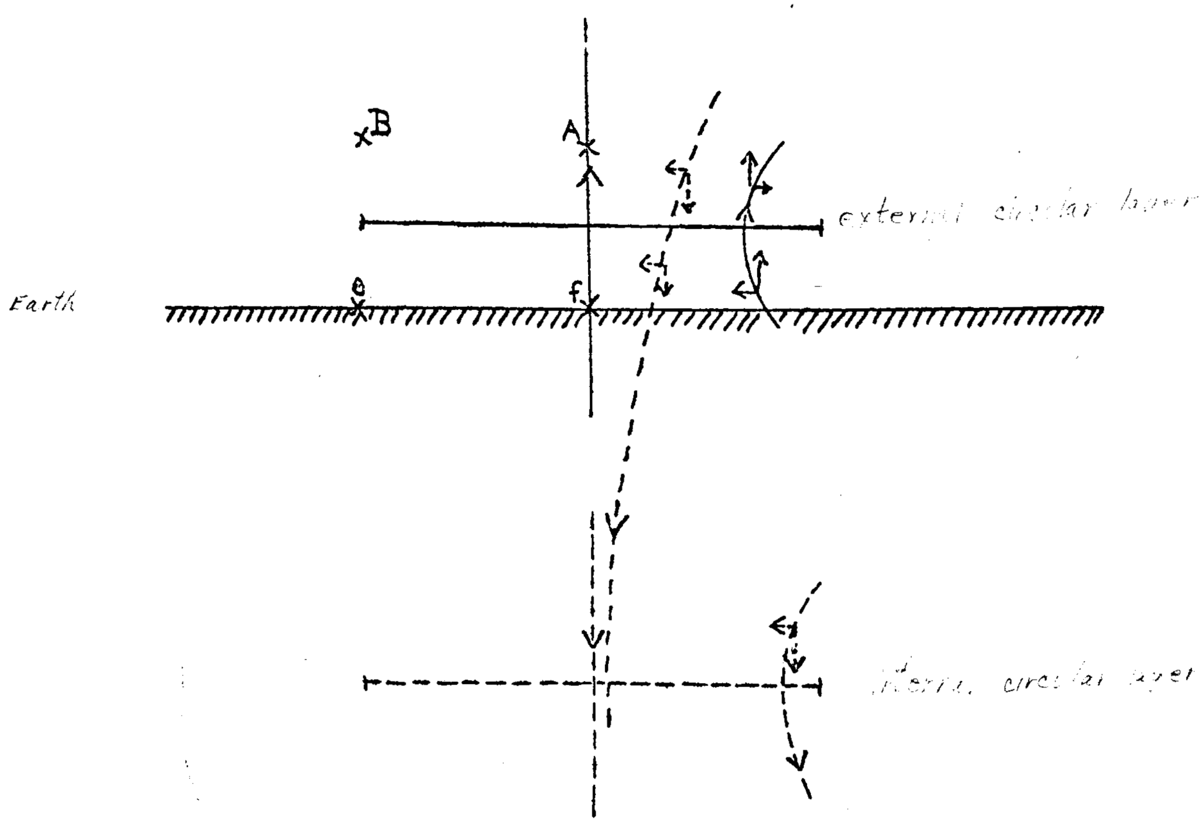


Figure 4.

But the geometry of the lines of force of circular layers is such that the component H changes sign from one side of the layer to the other. From /6
here on, this means that the effect on H of the induced currents is added to those of the inducing currents in the space included between the two systems.

A numerical example, drawn from the general results that will be presented below, will illustrate the importance of such an effect for the variation of the field with altitude. Let us assume that the induced system is at 500 km depth in relation to the earth, and the inducing system 100 km above, and let us consider a ΔZ equal to 100 gammas observed at f. By starting with an induction ratio of 40 percent, a generally accepted value, the effect at the earth of a single inducing system would be 166 gammas, that of the induced system--66 gammas. At a point A located 100 km above the inducing system, the effect due to this is again 166 gammas; the effect of the other system is no more than -56 gammas. The total field at A is therefore equal to 110 gammas; it is greater than at f, and keeps the same sign.

On the other hand, at a point such as e, the corresponding values of H would be, for 100 gammas observed at the earth, 71 gammas due to the inducing system and +29 gammas due to the induced system. And, at a point B 100 km above the inducing system:-71 gammas due to the inducing system, + 26 gammas due to the induced system, for an effective total of -45 gammas.

Consequently, when we pass from the earth at a point image of the other side of the external system the composition of the effects of the two systems is such that the field S is slightly increased along Z but changes sign and is practically divided by a factor 2 in absolute value along H.

It seems then that the decrease with altitude of the field S is profoundly modified by the combined effect of two systems with respect to the decrease due to a single system.

Figure 5 enables us to appreciate this fact. It is relative to the modulus of the total component f, each section of the figure corresponding to different values of the induction ratio between the two systems. /7

The current model used is that described in § 1 (see fig. 1), for a parabolic law of current density variation. The field has been calculated along verticals from the point e where $\vec{F} = \vec{H}$, along the vertical of the point f where $\vec{F} \# \vec{Z}$, and along verticals of two intermediate points with abscissas 1200 km and 2200 km.

In the approximation of the plane model that we are studying, these four points are roughly representative of the field S at the latitude of the equator, at the latitudes 12° and 22° and at the latitude of the focus.

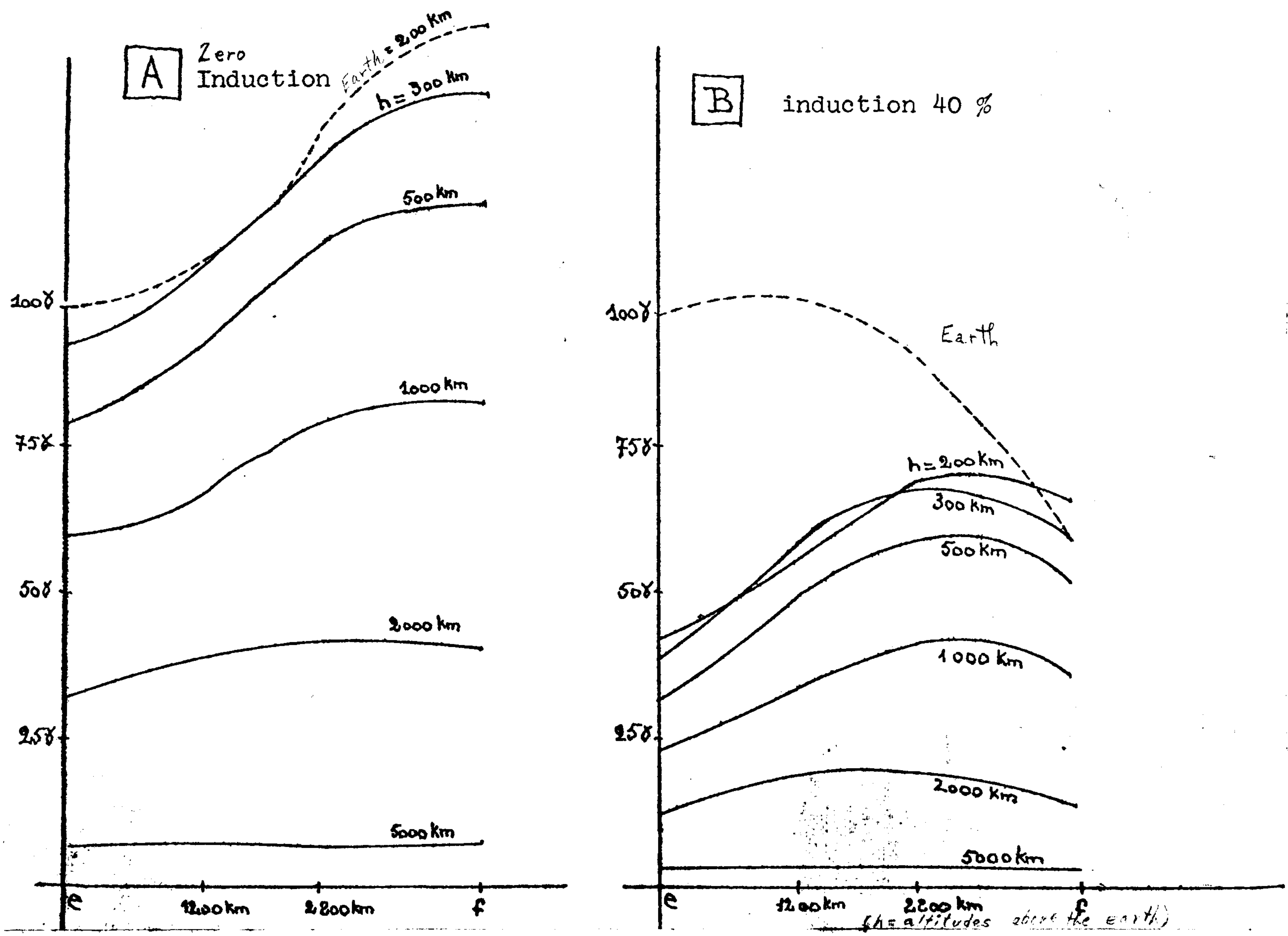


Figure 5. Values of the modulus of $\vec{F}_s$

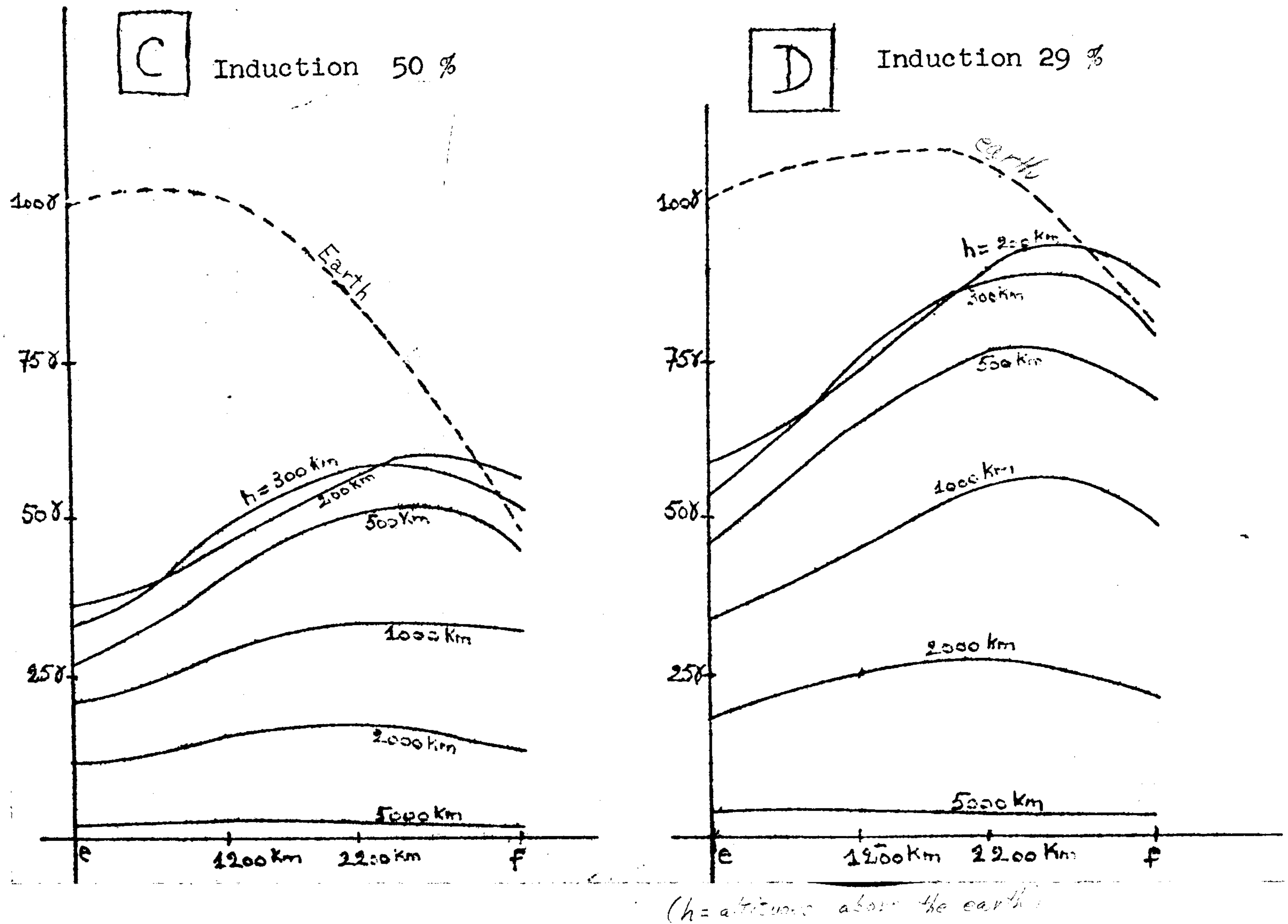


Figure 5 (continued).

Each of the curves represents the variation of $|\vec{F}_s|$ at an altitude given relative to the earth between these latitudes.

In A, where the induction is assumed to be zero, and which consequently represents the effect of a single system, the values at the earth (100 km below the system) are identical in absolute value to those at 200 km altitude relative to the earth (100 km above the system). Curves representing the variation of $|\vec{F}_s|$ at each point as a function of the altitude should be identical to those of figure 2 for the equator ($F = H$) and the focus($F \# Z$); they should be slightly different for the two intermediate latitudes.

$|\vec{F}_s|$ is the greatest, for low altitudes, at the latitude of the focus. The value 100 gammas, taken as the value on the earth at the equator, has been chosen because it is a convenient reference for evaluating the decrease; it represents the mean observed value of H, at tropical stations, at the time of the last maximum of solar activity.

In B, C and D different values of the induction ratio have been selected (40 percent, 50 percent, 30 percent) to show the importance of this factor relative to the decrease with altitude. A variation of the depth chosen for the induced system (500 km, or 600 km between the two systems) should /8
modify the results very little.

The comparison of the curves of B and A (an induction ratio of 40 percent is generally accepted for real Sq systems) is significant.

First of all, at the earth the variation of $|\vec{F}_s|$ is completely modified. While the ratio of the values of $|\vec{F}_s|$ at the focus and at the equator is about 1.47 in A, it becomes 0.63 with the induction effect. For several months during the summer of 1958 the ratio of the amplitudes of the diurnal variation of Z at Toledo (focus) and of H at M'Bour (# equator) was 0.55. The approximation obtained by our plane model is therefore quite good.

At altitudes, the curves analogous to those of figure 2 (which corresponded to a zero induction) should have a completely different form; figure 6 will permit a comparison. The values of the indicated abscissas represent altitudes above the earth, and no longer altitudes above the layer; to the left are shown the observed values at the earth for reference.

So it seems that the decrease with altitude of the field caused by two plane layers of circular currents is almost entirely determined by the induction factor. It is certainly the predominant characteristic.

A double question should then be posed: to what degree is the plane model representative of the real phenomenon, which is composed of current systems circulating in a sheet of hemispherical form? On the other hand, satellite measurements of the modulus of Fe do not reach the quantity $\left|\vec{F}_s\right|$ which has been studied in this paragraph, but only a quantity such that

$$\Delta F = \left|\vec{F}_o + \vec{F}_s\right| - \left|\vec{F}_o\right|$$

where $\vec{F}_o$ represents the value of the field in the absence of the field S. But the geometry of the lines of force of the field $\vec{F}_o$ is different from that /9
of the field $\vec{F}_s$, and this fact introduces another factor that can modify the preceding results.

3. Approximation of the Plane Model

Our objective is limited to estimating the decrease with altitude of the field S, in the vicinity of the meridian, that is, in the vertical plane containing one of the axes of symmetry (axis ff) of the model of figure 1.

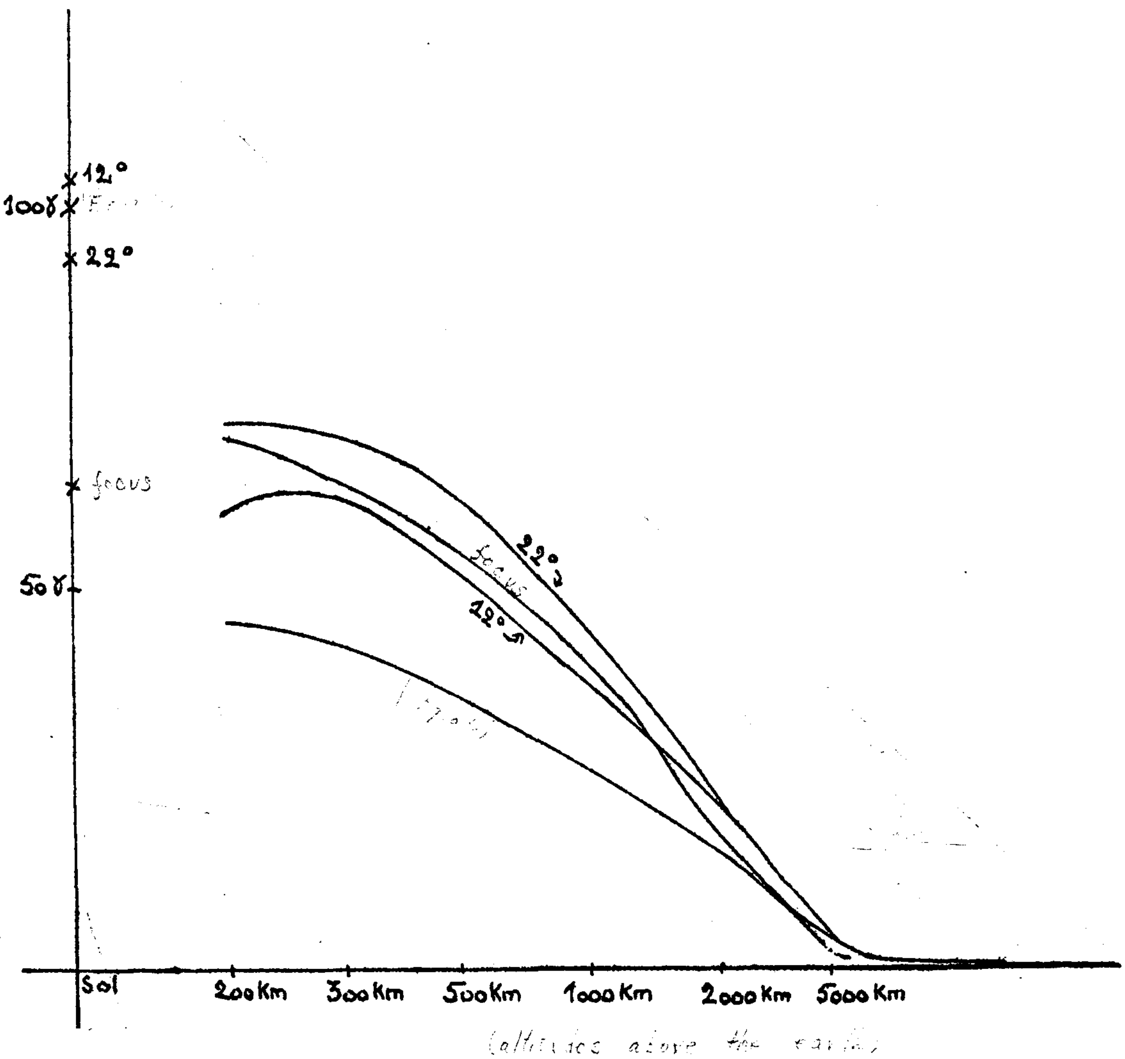


Figure 6. Variation with altitude of $|\vec{F}_s|$ for an induction ratio of 40 percent.

If we impose a curvature on the plane model in the direction of the line EE (let it still be in the direction of the longitudes), the vertical plane passing through ff remains a plane of symmetry of the model and the direction of the field is not changed. But the tubes of force, on the outside of the curvature imposed on the model, are distributed over a larger space; the intensity is therefore decreased by a factor inversely proportional to the increase in volume.

When we now impose a curvature on the model in the direction of the latitudes, reasoning similar to that applied above can be used from the first for the vertical located at e in the meridian plane. The direction of the field is not changed here, but the tubes of force are distributed in the elements of a volume which increase with altitude like the elements of volume included between two spheres of radius r and r + dr on the inside of a solid angle w. Such volume elements are proportional to $3\ wr^2dr$, and the increase of volume of these elements between a sphere of radius r_o and a sphere of radius r is itself proportional to r^2/r_o^2. Consequently, we can estimate that, on the vertical at e, a factor such as $\Delta v = r_o^2/(r_o + h)^2$ (r_o, radius of the earth; h, altitude) should give a rough idea of the approximation introduced by the use of a plane system for estimating the intensity of the field with altitude.

At points other than e the approximation introduced by the plane model is more difficult to estimate.

At low altitudes, the effect of nearby current elements is un- /10
doubtedly predominant, and consequently the plane model remains representative. But at high altitudes the effect due to the layers C_S tends to become important and is compounded with the effect of the layers C_N in a direction which is modified by the curvature imposed on the model. We can try to appreciate the importance of this effect by comparing, as a function of altitude the respective magnitudes of the vectors $\vec{F}_s$ due to each of the systems C_N and C_S.

Figure 7 represents the effects, for verticals located at points 1200 and 2200 km from the line ff of figure 1, the respective effects of the systems C_N (vectors in solid lines) and C_S (vectors in dashed lines) in the case of zero induction and a plane model. We can conclude that the effect of the system C_S

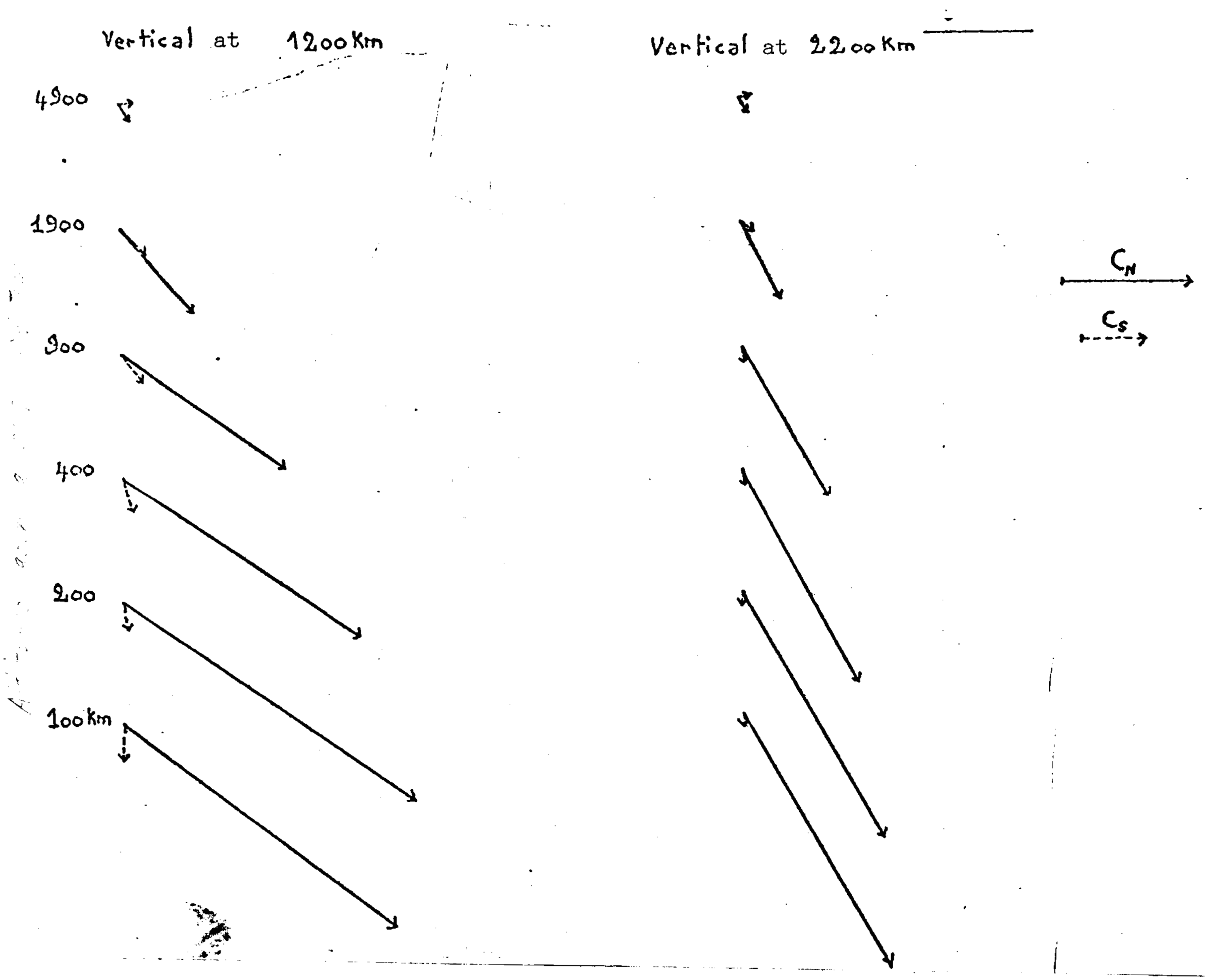

Vertical at 1200 Km
Vertical at 2200 Km
4900
1900
900
400
200
100 km
C_N
C_S

Figure 7.

is practically negligible on the vertical at 2200 km (and therefore a fortiori on the vertical of the focus).

This is not so on the vertical at 1200 km. However, a maximum is the order of magnitude of the change caused by a curvature imposed on the plane model can be estimated by assuming, for example, that the normals to the systems C_N and C_S form an angle of 70° (the difference between latitudes equivalent to the points f of figure 1). We should then rotate the vectors C_S by 70° in a clockwise direction. In fact, such an effect is in part compensated since the points of the vertical at 1200 km are sufficiently out of center in the system C_N for the effect to which they are most sensitive; and the curvature in the opposite direction of the system C_N to the north of this vertical certainly produces a more important effect in the opposite direction.

Finally, we can consider that the vectors $\vec{F}_s$ obtained in the preceding paragraph, starting with a plane model, are still representative when we impose on this model a curvature in the direction of the latitudes and as we bring the direction of these vectors to the new direction of the vertical. On the other hand, from the point of view of the intensity, factors such as Δv should have a similar significance. /11

4. Estimate of $\Delta F = \left|\vec{F}_o + \vec{F}_s\right| - \left|\vec{F}_o\right|$

Taking as a model of the field $\vec{F}_o$ the field of the dipole on a geographical meridian where the geomagnetic declination is zero, we have calculated the algebraic values of ΔF for the case of an induction ratio of 40 percent with the same base of 100 gammas observed on the earth at the equator and by assuming the layers C_N and C_S to be symmetrical with respect to the geomagnetic equator. Figure 8 shows the results obtained for each altitude as a function of the

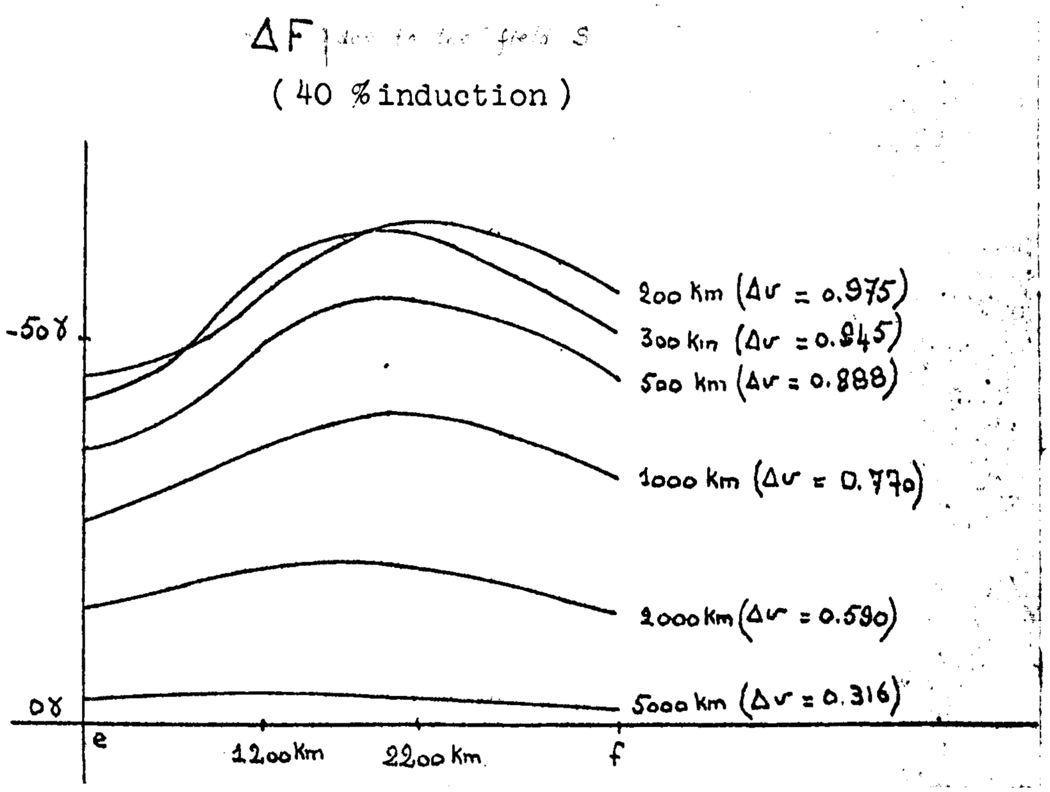


Figure 8.

position of the verticals in the system of currents. Factors Δv are indicated for each altitude. They give an order of magnitude of the volume effect estimated in the preceding paragraph. The ordinates of each curve should be roughly multiplied by these factors to take account of them.

The ΔF progress negatively toward the top in order to make comparison with figure 5B easier; they are not very different from $|\vec{F}_S|$. The latitude region included between the equator and the focus should then clearly become that which makes most susceptible the observation of the ΔF at the greatest altitudes. A latitude displacement of the order of 10^o of the systems C_S and C_S with respect to the geomagnetic equator should introduce a variation of ΔF of the order of ±10 gammas at an altitude of 200 km. Beyond the focus, toward higher latitudes, we can see that, since the H_S become positive with altitude the ΔF will continue to decrease.

5. Effect of the Electrojet

We have tried to estimate the effect due to the equatorial electrojet by calculating the effects with altitude of a band of constant current of 600 km width located 100 km above the earth and of another band of the same width located at a depth of 500 km, the induction ratio being 40 percent. The principal approximation thus introduced is that of a constant current. /12 In reference 4, Forbush and Casaverde have shown that it holds good. On the other hand, the real width of the electrojet varies without doubt from 660 km (Peru) to 450 km (Africa).

Figure 9 (similar to figure 5 for the case of an induction ratio of 40 percent) shows the variation on the earth and with altitude of the modulus of $\vec{F}_j$ caused by such a band, as a function of latitude. The base of 100 gammas on the earth at the equator has the same significance as that taken for the field $\vec{F}_S$. It represents the value observed at the time of the last maximum of solar activity; and since the mean amplification of Sq by the electrojet is about 2, we will be able to directly compare and add the values $\vec{F}_j$ to the values $\vec{F}_s$.

Compared to part B of figure 5, we see that the effect of the electrojet decreases more rapidly with altitude than that of the field S. It is still important in a band about 1000 km or both sides of the equator.

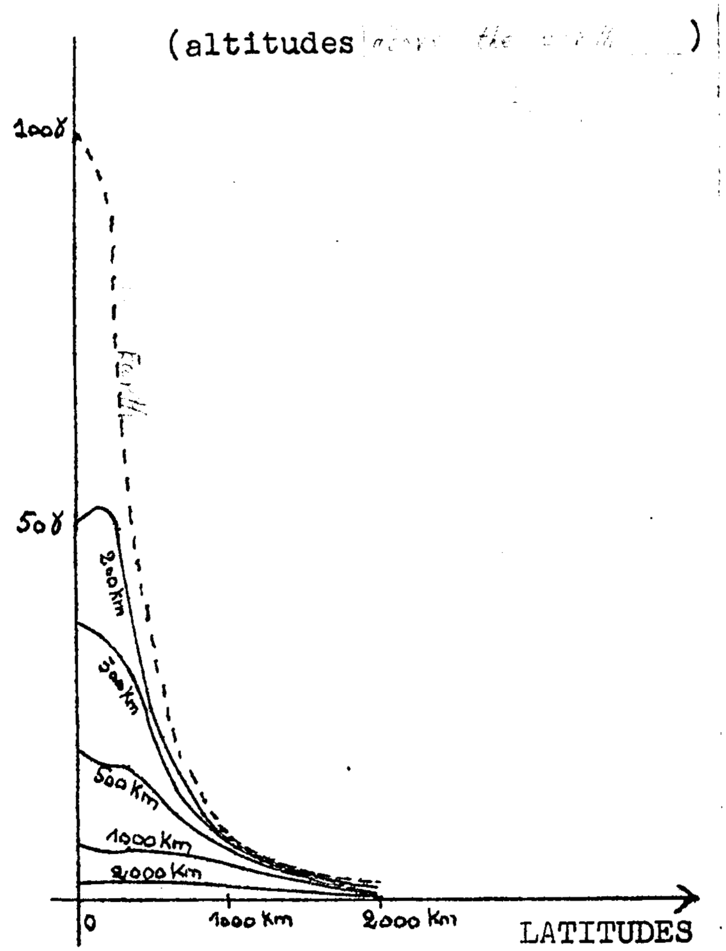


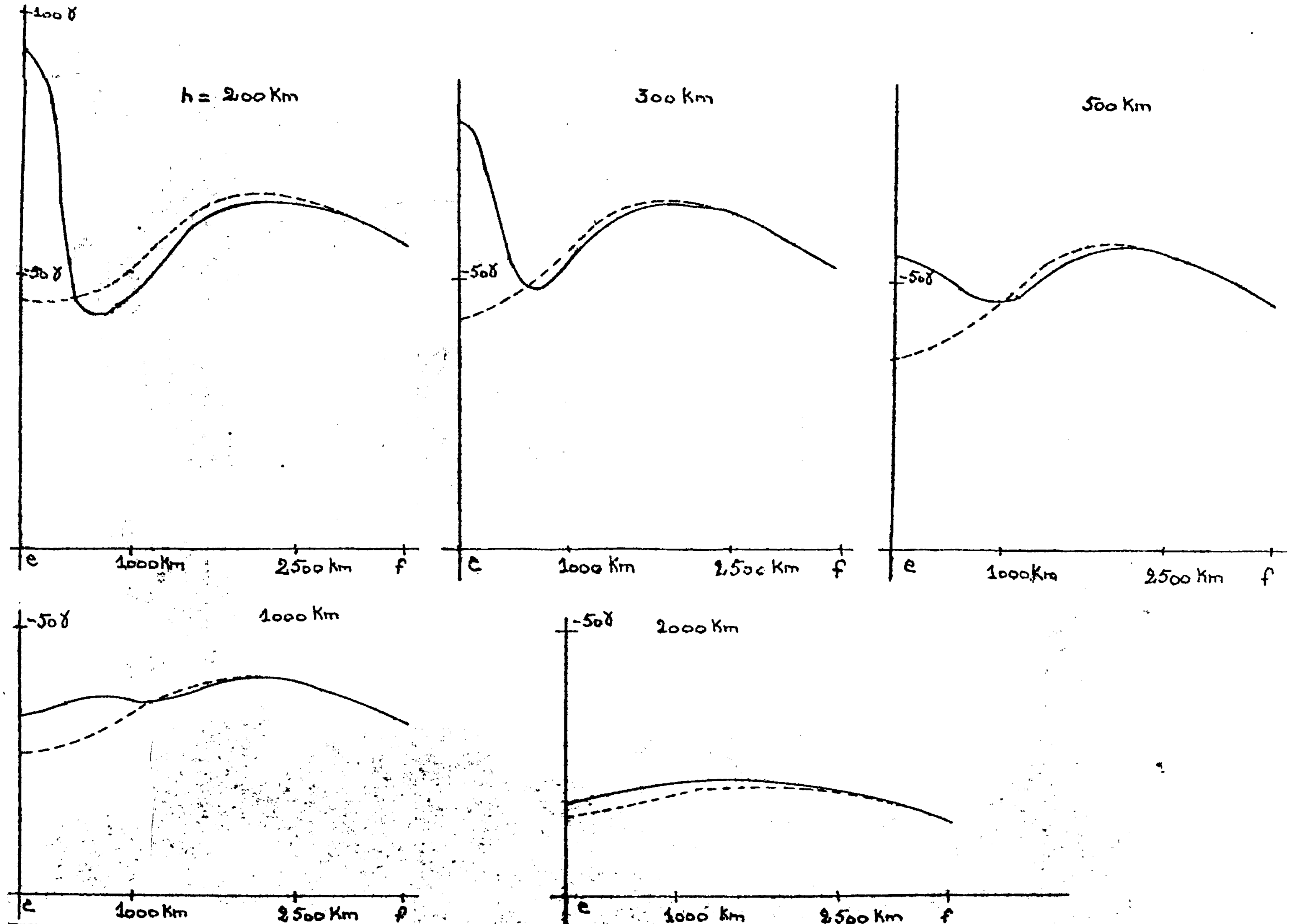
Figure 9. Values of the Modulus of $\vec{F}$.

Figure 10 is similar to figure 8. It represents

$$\Delta F = \left|\vec{F}_o + \vec{F}_s + \vec{F}_j\right| - \left|\vec{F}_o\right|$$

going negatively toward the top. The dashed parts of the curve represent the ΔF due to $\vec{F}_s$ alone.

At low altitudes ΔF thus appears to be decreased by the effect of the electrojet in a band of intermediate latitudes. This is due to the fact that, by deviating from the electrojet, the internal band becomes rapidly preponderant for the horizontal component with respect to the external band because its distance to the considered altitudes is greater (this characteristic does



ΔF due to the field $\vec{S} + \vec{J}$

Figure 10.

not appear in figure 3, relative to F_j). The ΔH_j therefore becomes positive and opposite to the ΔH_S. The effect rapidly disappears with altitude.

We can say roughly that the effect of the electrojet is important, in ΔF, on a band of 1000 km on both sides of the equator. On the equator, it doubles the ΔF at 200 km altitude; it increases it by a third at 500 km; at 2000 km it is negligible relative to the field S. /13

6. Conclusion

Measurements at altitude made on the modulus of $\vec{F}$ alone enable us to consider all transient variation of the field, in a direction perpendicular to the mean field F_o to be negligible: for a field F_o = 30,000 gammas, a perpendicular variation equal to 250 gammas should produce a ΔF of only one gamma. The effects observed at the earth on the component D are therefore practically negligible, at altitude in a measurement of $\vec{F}$. This is the reason that we limit ourselves to an estimate of the effect of the fields $\vec{S}$ and $\vec{J}$ uniquely in the plane of symmetry of the systems C_N and C_S perpendicular to the equator, corresponding to a meridian plane in the vicinity of the noon meridian.

Just as elsewhere, the diurnal intensity variation very rapidly culminates around noon (it is reduced by nearly one half on H or Z, three hours before or after its maximum), we can consider that curves such as those of figure 10 represent an order of magnitude of the maximum of the ΔF caused by the fields $\vec{S}$ and $\vec{J}$. Two other factors still reduce the importance of this effect: on one hand the volume effect of which the quantities Δv of figure 8 give an order of magnitude and on the other hand the fact that the base 100 gammas that was chosen as a reference corresponds to the maximum effect observed at the time of the last solar cycle.

Can we imagine treating the effects due to the fields $\vec{S}$ and $\vec{J}$ as background noise?

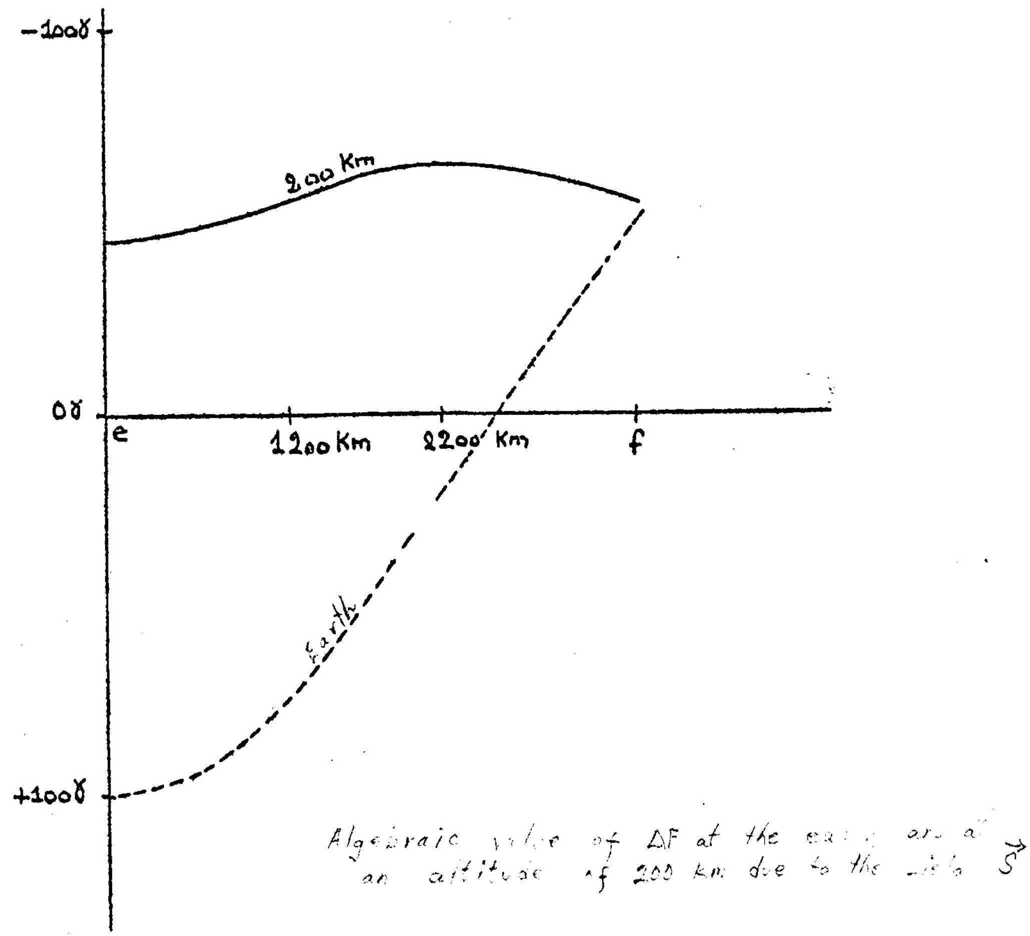


Figure 11.

Figure 11 represents, for the field S alone, the algebraic F (positive toward the base) at the earth and at an altitude of 200 km as a function of the latitude. At the earth, the gradient of the variation is important (+100 γ at -50 γ); at altitude it is weak (1/20 γ). At latitudes higher than that of the focus, the two curves should diverge anew:[1] the ΔF at the earth /14

[1]In fact, at altitude, the ΔH_s become positive while the ΔZ stay negative; the resultant direction of $\vec{F}_s$ therefore makes a greater angle with that of the vector $\vec{F}_o$ than the direction of $\vec{F}_s$ at the earth (where ΔH_s becomes negative, and ΔZ_s remains negative.

first becomes greater in absolute value before tending to zero; ΔF at altitude continues to decrease in absolute value. Moreover, they tend toward zero more rapidly than $\vec{F}_s$ since the angle between $\vec{F}_o$ and $\vec{F}_s$ tends toward $\pi/2$.

Thus, the situation seems infinitely more favorable at altitude than at the earth. On one hand, the background noise produced at altitude by ΔF has an amplitude that is practically half that which exists at the earth. On the other hand, and most important, from the abstraction made of the effect of the field $\vec{J}$ (which causes a rapid gradient at altitudes just as at the earth and for which a reasonable model is certainly easier to obtain than for the field $\vec{S}$), the ΔF at altitude show, as a function of latitude, a relatively slow variation. From this fact, models that are still relatively simple[1] but are more complex than those that we have used should, without doubt, enable us to obtain a reasonable approximation for a reduction intended to provide knowledge of the field $\vec{F}_o$ (project POGO, for example). This is particularly so for models that take account of the frequent dissymmetry between the two hemispheres. In such a model, an error made in the real latitude of the systems $\vec{S}$ should have little influence.

In the case of project M A G N O S (ref. 5) figure 10 shows that, at an altitude of 500 km, scanning some $\pm 20^{\circ}$ or 30° of latitude should cause only a small variation in ΔF.

Finally, a last remark can be made. Figure 11 shows that at the latitude of the focus ΔF is practically the same at the earth and at 200 km altitude: therefore, rockets should permit detection of the currents Sq only by a comparison between day flights and night flights. Equatorial latitudes are

[1] It could possibly suffice to base them on an estimate of the ΔH observed at the earth at the noon meridian.

obviously most favorable; but tropical latitudes should still be quite sufficient. In particular, the determination of the altitude and thickness of the layer /15 should be made by observing the altitude zone in which the change of sign ΔF occurs. On both sides of this zone one should observe a constant ΔF since the approximation of a layer of infinite width is being used.

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